Noncritical M-theory and the Gross-Neveu model in $2+1$ dimensions

Anastasios C. Petkou[*]

Department of Physics,
University of Crete,
71003 Heraklion, GREECE.

George Siopsis[†]

Department of Physics and Astronomy,
The University of Tennessee, Knoxville,
TN 37996 - 1200, USA.

June 2006

Abstract

We point out a non-trivial connection between the model proposed by Hořava and Keeler as a candidate for noncritical M-theory and the Gross-Neveu model with fermionic fields obeying periodic boundary conditions in $2+1$ dimensions. Specifically, the vacuum energy of the former is identified with the large-N free-energy of the latter up to an overall constant. This identification involves an appropriate analytic continuation of the subtraction point in noncritical M-theory, which is related to the volume of the Liouville dimension. We show how the world-sheet cosmological constant may be obtained from the Gross-Neveu model. At its critical point, which is given in terms of the golden mean, the values of the vacuum energy and of the cosmological constant are 4/5 and 2/5 of the corresponding values at infinite string coupling constant.

---

[*]E-mail: petkou@physics.uoc.gr
[†]E-mail: siopsis@tennessee.edu

1   Introduction

Non-relativistic fermi liquid models have been extensively used in the study of non-critical string theories in two dimensions [1–14]. In an interesting recent work [15] it has been suggested that fermi liquid model might also be used in the formulation of noncritical M-theory in three dimensions. The authors of [15] propose to identify the vacuum energy of two-dimensional non-relativistic fermions in an inverted harmonic oscillator potential with the energy of noncritical M-theory in $2 + 1$ dimensions.

In this note, we show that the exact vacuum energy of the fermi liquid system calculated in [15] and the large-N free energy of a three-dimensional Gross-Neveu model are given by remarkably similar expressions after an appropriate analytic continuation of the subtraction point needed to define the density of states in the fermi liquid system. In order to make the identification, the fermionic fields of the model must be given periodic boundary conditions along the compactified dimension. This non-trivial relation points to an interesting correspondence between the two models.

2   Noncritical M-theory

We start with a discussion of the salient features of the $2 + 1$-dimensional model proposed by Hořava and Keeler [15] aiming at casting the main results in a form appropriate for comparison with the three-dimensional Gross-Neveu model.

In [15] noncritical M-Theory in $2 + 1$ dimensions can be described by the double-scaling limit of non-relativistic fermionic fields $\psi(x^0, \bar{x}^1, \bar{x}^2)$ with action [15]

$$I_M = \int dx^0 \int d^2\bar{x} \left(\psi^\dagger \partial_0 \psi - \tfrac{1}{2} \partial_i \psi^\dagger \partial^i \psi + V(\bar{x}) \psi^\dagger \psi \right) , \tag{1}$$

where the potential is

$$V(\bar{x}) = \tfrac{1}{2} \omega_0^2 \bar{x}^2 + \dots . \tag{2}$$

The semiclassical density of states is obtained from the single particle Hamiltonian

$$H(\bar{p}, \bar{x}) = \tfrac{1}{2} \bar{p}^2 - V(\bar{x}) . \tag{3}$$

The system is equivalent to a two-dimensional inverted harmonic oscillator in the scaling limit of interest. The density of states can be defined in terms of the resolvent

$$\rho(\mu) \equiv \frac{1}{\pi} \Im \mathrm{Tr} \frac{1}{H + \mu - i\epsilon} = \frac{1}{\pi} \Re \int_0^\infty d\tau \, e^{-i(\mu - i\epsilon)\tau} \mathrm{Tr} e^{-i\tau H} \tag{4}$$

It may also be obtained by summing the type-0A contributions of all sectors with integer values of the RR flux [15].

The partition function $Z \equiv \mathrm{Tr} e^{-i\tau H}$ appearing in (4) can be derived from the partition function Z_2 of a normal two-dimensional harmonic oscillator by continuing to imaginary frequency [11]. [1] Namely we consider

$$Z_1 = \sum_{n=0}^\infty e^{-\tau(n+\frac{1}{2})\omega} , \tag{5}$$

---

[1]It should be pointed out that one ought to exercise caution in performing such an analytic continuation. In the analogous two-dimensional case, justification for its validity may be found in [16]. The calculation appies to the present three-dimensional case with little modification.

and we obtain

$$Z_2 = Z_1^2 = \frac{1}{4\sinh^2 \frac{\omega\tau}{2}}. \tag{6}$$

To apply this result to the inverted two-dimensional harmonic oscillator, we rotate $\omega \to -i\omega_0$ and $\tau \to i\tau$ and we deduce

$$Z \equiv \mathrm{Tr}\, e^{-i\tau H} = \frac{1}{4\sinh^2 \frac{\omega_0\tau}{2}}. \tag{7}$$

Then the density of states (4) reads

$$\rho(\mu) = \frac{1}{4\pi} \Re \int_0^\infty d\tau\, e^{-i\mu\tau} \frac{1}{\sinh^2 \frac{\omega_0\tau}{2}}. \tag{8}$$

The integrand has a singularity as $\tau \to 0$ which can be regulated in a Pauli-Villars manner, by introducing a subtraction point M and defining

$$\rho_M = \rho(\mu) - \rho(M). \tag{9}$$

The UV cutoff M is related to the volume of the Liouville field in string theory. To calculate the integral, we bring it into the form

$$\rho_M = \frac{1}{2\pi} \int_0^\infty d\tau\, \frac{\sin^2 \frac{\mu\tau}{2} - \sin^2 \frac{M\tau}{2}}{\sinh^2 \frac{\omega_0\tau}{2}}. \tag{10}$$

Making use of [17]

$$\int_0^\infty dx\, \frac{\sin^2 \beta x}{\sinh^2 \pi x} = \frac{\beta}{\pi(e^{2\beta}-1)} + \frac{\beta-1}{2\pi}, \tag{11}$$

we obtain

$$\rho_M = -\frac{\mu}{2\pi\omega_0^2} \coth \frac{\pi\mu}{\omega_0} + \frac{M}{2\pi\omega_0^2} \coth \frac{\pi M}{\omega_0}. \tag{12}$$

As $M \to \infty$, this is well approximated by

$$\rho_M = -\frac{\mu}{2\pi\omega_0^2} \coth \frac{\pi\mu}{\omega_0} + \frac{M}{2\pi\omega_0^2}, \tag{13}$$

in agreement with the result of [15]. Instead of (13), we shall use the more general form (12), which is valid for arbitrary values of M, in order to compare with the Gross-Neveu model.

The vacuum energy is given by

$$F = \int^{-\mu} d\mu'\, \mu'\, \rho_M(\mu') \tag{14}$$

Integrating, we obtain

$$F(\mu) = \frac{1}{2\pi^3\beta} \left\{ \frac{M\beta^3\mu^2}{8} \coth \frac{\beta M}{2} - \frac{\beta^3\mu^3}{12} - \frac{\beta^2\mu^2}{2} \ln(1 - e^{-\beta\mu}) + \beta\mu\, \mathrm{Li}_2(e^{-\beta\mu}) + \mathrm{Li}_3(e^{-\beta\mu}) \right\} \tag{15}$$

where we introduced the length scale

$$\beta = \frac{2\pi}{\omega_0}, \tag{16}$$

3

in order to facilitate comparison with the Gross-Neveu model. The latter will be identified with the length of the finite dimension of the three-dimensional Euclidean space on which the Gross-Neveu model is defined.

To make contact with string theory one may introduce the conjugate variable Δ (worldsheet cosmological constant) defined as

$$dF = \mu d\Delta \,. \tag{17}$$

Then the vacuum energy should be viewed as a function of Δ ($F = F[\mu(\Delta)]$). One easily obtains

$$\Delta(\mu) = \frac{1}{4\pi^3} \left\{ \frac{M\beta^2\mu}{2} \coth\frac{\beta M}{2} - \frac{\beta^2\mu^2}{4} - \beta\mu \ln(1 - e^{-\beta\mu}) + \mathrm{Li}_2(e^{-\beta\mu}) \right\} \,. \tag{18}$$

In the large-μ limit, which may be thought of as corresponding to weak coupling, [2] we have

$$F(\mu) \approx -\frac{\beta^2\mu^3}{24\pi^3} \;\;, \;\;\; \Delta(\mu) \approx -\frac{\beta^2\mu^2}{16\pi^3} \,, \tag{19}$$

so that $F \sim \Delta^{3/2}$, to be compared with the string theory result $F \sim \Delta^2/\ln\Delta$. Not only is the logarithm absent, we also have a string susceptibility exponent [3–7] $\gamma_{\mathrm{str}} > 0$, where $F \sim \Delta^{2-\gamma_{\mathrm{str}}}$, indicating that we have crossed the string barrier of central charge $c \leq 1$.

In the $\mu \to 0$ limit, which analogously may be thought of as corresponding to strong coupling, we obtain

$$F(0) = \frac{1}{2\pi^3}\frac{1}{\beta}\mathrm{Li}_3(1) = \frac{\zeta(3)}{2\pi^3\beta} \;\;, \;\;\; \Delta(0) = \frac{1}{4\pi^3}\mathrm{Li}_2(1) = \frac{1}{24\pi} \,, \tag{20}$$

a behavior reminiscent of an asymptotically free theory [11]. For comparison, in string theory the vacuum energy approaches a finite constant whereas $\Delta \to 0$. Notice also that the UV regulator M contributes in neither limit ($\mu \to 0$, $\mu \to \infty$).

3   The Gross-Neveu model

Next, we consider the Gross-Neveu model in three Euclidean dimensions, one of which has finite length β. The action is [18–21]

$$I_{GN} = -\int_0^\beta dx^0 \int d^2\bar{x} \left(\bar{\psi}^a \slashed{\partial}\psi^a + \frac{g}{2}(\bar{\psi}^a\psi^a)^2 \right) \,, \tag{21}$$

where $\bar{\psi}^a$, ψ^a ($a = 1, 2, \ldots, N$) are here taken to be two-component Dirac fermions satisfying periodic boundary conditions along the finite dimension.[3] The γ-matrices are defined in terms of the Pauli matrices as $\gamma^i = \sigma^i$ ($i = 1, 2$) and $\gamma^0 = \sigma^3$.

The path integral of the model gives the subtracted partition function as

$$\begin{aligned}
Z &= e^{-\frac{N}{2}\beta[F(\beta) - F(\infty)]} \\
&= \int (\mathcal{D}\bar{\psi})(\mathcal{D}\psi)(\mathcal{D}\sigma) \exp\left\{ \int_0^\beta dx^0 \int d^2\bar{x} \left(\bar{\psi}^a(\slashed{\partial} + \sigma)\psi^a - \frac{1}{2g}\sigma^2 \right) \right\} \\
&= \int (\mathcal{D}\sigma) \exp\left\{ N\left(\mathrm{Tr}_\beta \ln(\slashed{\partial} + \sigma) - \frac{1}{2G}\int_0^\beta dx^0 \int d^2\bar{x}\,\sigma^2 \right) \right\} \,,
\end{aligned} \tag{22}$$

---

[2]Recall that in string theory $\mu \sim g_s^{-1}$.

[3]Periodic boundary conditions may be obtained by coupling the fermions to an external $U(1)$ gauge potential which twists the antiperiodic boundary condition of the fermions to generically anyonic ones [21, 22].

where F denotes the free energy, V_2 denotes the spatial volume and $G = gN$ is the rescaled coupling which is held fixed as $N \to \infty$ and $g \to 0$. In the second line of (22) we have introduced the auxiliary scalar field σ (with periodic boundary conditions) and in the third line we have integrated out the fermions.

We can evaluate the last path integral in (22) by a saddle point expansion setting[4] $\sigma = \mu + \lambda/\sqrt{N}$. Standard manipulations then give the leading-N result

$$
\begin{aligned}
Z &= \exp\left\{ N\beta V_2 \left(\frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^2\bar{p}}{(2\pi)^2} \ln(\bar{p}^2 + \omega_n^2 + \mu^2) - \frac{\mu^2}{2G} \right) \right\} \\
&\times \int (\mathcal{D}\lambda) \exp\left\{ \sqrt{N} \int_0^\beta dx^0 \int d^2\bar{x}\, \lambda(x) \left(\frac{2}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^2\bar{p}}{(2\pi)^2} \frac{1}{\bar{p}^2 + \omega_n^2 + \mu^2} - \frac{\mu}{G} \right) \right\}, (23)
\end{aligned}
$$

where, due to the periodic boundary conditions we have

$$
\omega_n = \frac{2\pi n}{\beta}\,, \qquad n = 0, \pm 1, \pm 2, \dots \tag{24}
$$

From (23) we read the large-N free energy F defined in (22),

$$
\frac{F(\beta)}{V_2} = \frac{\mu^2}{2G} - \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^2\bar{p}}{(2\pi)^2} \ln(\bar{p}^2 + \omega_n^2 + \mu^2) + \int \frac{d^3p}{(2\pi)^3} \ln p^2\,, \tag{25}
$$

and the gap equation ($\frac{\partial F}{\partial \mu} = 0$),

$$
\begin{aligned}
\frac{\mu}{G} &= \frac{2}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^2\bar{p}}{(2\pi)^2} \frac{1}{\bar{p}^2 + \omega_n^2 + \mu^2} \\
&= \frac{\mu}{G_*} - \frac{\mu^2}{2\pi} - \frac{\mu}{\pi\beta} \ln\left(1 - e^{-\beta\mu}\right) + O(\Lambda^{-1})\,. \tag{26}
\end{aligned}
$$

To arrive at the second line of (26) we used a large momentum cutoff Λ and denoted

$$
\frac{1}{G_*} = \int^\Lambda \frac{d^3p}{(2\pi)^3} \frac{1}{p^2}\,. \tag{27}
$$

The expression (25), up to an overall negative sign, coincides with the large-N free energy density of the three-dimensional $O(N)$ vector model [20] and the same applies also to the gap equation (26) up to the overall μ factor. In the $O(N)$ vector model, $-F(\beta)$ in (25) is interpreted as a physical free energy. In the case at hand, (25) can be a positive quantity and may be interpreted as energy.

To proceed, let us define a critical parameter $\mathcal{M}$, with dimensions of mass as

$$
\frac{\mathcal{M}}{4\pi} = \frac{1}{G_*} - \frac{1}{G} \sim \Lambda^2(G - G_*)\,. \tag{28}
$$

This will be kept finite (zero, negative or positive) in the scaling limit $\Lambda \to \infty$ and $G \to G_*$. $\mathcal{M}$ essentially quantifies the distance of $1/G$ from a critical value $1/G_*$ and determines the parameter μ by

$$
2\sinh\frac{\mu\beta}{2} = e^{\mathcal{M}\beta/2}\,. \tag{29}
$$

---

[4]We use on purpose the same notation μ here and in Section 2 to facilitate the comparison of results.

The parameter μ is the mass of the effective $\langle \lambda\lambda \rangle$ propagator, or alternatively it may be viewed as an inverse correlation length $\xi \sim 1/\mu$. In the $O(N)$ vector model at zero temperature, it is an order parameter for the $O(N) \to O(N-1)$ symmetry breaking [19].

If we substitute $1/G$ from the gap equation (26) into (25) and take the $\Lambda \to \infty$ limit, we find after some straightforward manipulations,

$$F(\beta) = \frac{V_2}{\pi\beta^3} \left\{ -\frac{\beta^3\mu^3}{12} - \frac{\beta^2\mu^2}{2} \ln(1 - e^{-\beta\mu}) + \beta\mu \operatorname{Li}_2(e^{-\beta\mu}) + \operatorname{Li}_3(e^{-\beta\mu}) \right\} . \quad (30)$$

Crucial in obtaining (30) was the fact that the linear divergence in $1/G_*$ canceled the linear divergence of the integral containing the logarithms in (25).

Comparing (30) with the result (15) for the vacuum energy in the Hořava-Keeler model, we observe a remarkable non-trivial correspondence provided we identify the length scale β with the corresponding one in the Hořava-Keeler model (eq. (16)) and the inverse correlation length μ with the opposite of the fermi energy in the fermi liquid of the Hořava-Keeler model. The identification of length scales amounts to identifying the harmonic oscillator frequency ω_0 in the Hořava-Keeler model with the fundamental frequency ω_1 of the Gross-Neveu model (24) as

$$\omega_0 = \omega_1 = \frac{2\pi}{\beta} . \quad (31)$$

The two expressions (30) of the Gross-Neveu model and (15) of the Hořava-Keeler model differ in two respects.

(a) In an overall constant dimensionless factor coming from the ratio V_2/β^2, and

(b) to obtain agreement, the subtraction point needed to define the vacuum energy (15) ought to be analytically continued to the imaginary value

$$M = \frac{(2n+1)\pi i}{\beta} \ , \quad n \in \mathbb{Z} \quad (32)$$

The free energy (30) should be viewed as a function of the critical parameter $\mathcal{M}$ (29). The latter is conjugate to the parameter μ^2; at fixed temperature we have

$$\frac{dF}{V_2} = -\mu^2 \frac{d\mathcal{M}}{4\pi} , \quad (33)$$

to be contrasted with the Legendre equation (17) defining the world-sheet cosmological constant Δ in the Hořava-Keeler model. We may similarly obtain a two-dimensional "cosmological constant" in the Gross-Neveu model starting from the expression (30) for the free energy. Instead of introducing a Legendre transform for μ^2, let us define $\mathcal{D}$ by conjugating with respect to μ, i.e.,

$$dF = \frac{\omega_0^2 V_2}{2} \mu d\mathcal{D} . \quad (34)$$

A short calculation yields

$$\mathcal{D} = \frac{1}{4\pi^3} \left\{ \operatorname{Li}_2(e^{-\beta\mu}) - \frac{\beta^2\mu^2}{4} - \beta\mu \ln(1 - e^{-\beta\mu}) \right\} , \quad (35)$$

which agrees with the the Hořava-Keeler model parameter Δ (18) for the special choice (32) of the subtraction point M.

6

In the limit $\mu \to 0$, we obtain

$$F(\beta) = \frac{V_2}{\pi \beta^3} \, \zeta(3) \ , \quad \mathcal{D} = \frac{1}{24\pi} \, . \tag{36}$$

Notice that $F(\beta)$ reduces to the free energy of a free fermion and $\mathcal{D}$ to the strong-coupling expression for the cosmological constant (20).

4   Discussion

We have shown that the fermi liquid model of noncritical M-theory introduced in [15] yields an expression for the vacuum energy which agrees (up to an overall multiplicative constant) with the expression for the free energy one obtains in the Gross-Neveu model, after analytical continuation of the subtraction point M (which is related to the Liouville field volume) to one of the values (32). To achieve agreement, we had to make two identifications:

(i) the harmonic oscillator frequency ω_0 was identified with the fundamental frequency ω_1 of the Gross-Neveu model (eq. (31)), and

(ii) the opposite of the fermi energy in the fermi liquid model was identified with the inverse length scale μ (mass pole of the two-point function of the auxiliary field σ) of the Gross-Neveu model.

We could then obtain string dynamics in the Gross-Neveu model as in the Hořava-Keeler model by introducing a Legendre transform with respect to μ. However, one normally thinks of the critical parameter $\mathcal{M}$ in the Gross-Neveu model in terms of a Legendre transform with respect to μ^2, which yields the gap eq. (26). For $\mathcal{M} > 0$ and fixed finite β, we cannot go to the $\mu \to 0$ limit. Instead, we obtain a non-trivial critical point in the scaling limit $\Lambda \to \infty$ and $\mathcal{M} \to 0$, which corresponds to a non-trivial three-dimensional CFT [20]. Solving the gap equation (26) for $\mathcal{M} = 0$, we obtain the critical value

$$\mu \equiv \mu_* = \frac{2}{\beta} \, \ln \tau \ , \quad \tau = \tfrac{1}{2}(1 + \sqrt{5}) \, , \tag{37}$$

given in terms of the golden mean. The free energy at the critical point is

$$F(\beta; \mu_*) = \frac{4}{5} \frac{V_2}{\pi \beta^3} \, \zeta(3) \, . \tag{38}$$

i.e., $4/5$ of its value at $\mu = 0$ (free fermion). This is the result of non-trivial polylogarithm identities (see Appendix A). The same rational relationship holds for the vacumm energy in the Hořava-Keeler model, if we allow M to take on a value given by eq. (32). We deduce from (15),

$$F(\mu_*) = \frac{4}{5} \, F(0) \, . \tag{39}$$

Moreover, the world-sheet cosmological constant at the critical point $\mu = \mu_*$ is related to its value at $\mu = 0$ (strong coupling limit) by

$$\Delta(\mu_*) = \frac{2}{5} \, \Delta(0) \, , \tag{40}$$

which is obtained from (18) for M given by (32) and using dilogarithm identities (see Appendix A). It would be interesting to understand what two-dimensional critical model this critical point might correspond to.

Finally, it would be of great interest to investigate the possibility of extending the observed correspondence between noncritical M-theory and the Gross-Neveu model in three dimensions to correlation functions. We hope to report on progress in this direction in the near future.

Acknowledgments

A   Polylogarithm identities

Here we present certain polylogarithm identities which we used in order to derive eqs. (39) and (40). For more details see [24].

The nth-order polylogarithm is defined by

$$\mathrm{Li}_n(x) = \sum_{k=1}^{n} \frac{x^k}{k^n} \tag{A.1}$$

We have $\mathrm{Li}_n(1) = \zeta(n)$.

The dilogarithm obeys the following identities:

$$\mathrm{Li}_2(x) + \mathrm{Li}_2(1-x) = \mathrm{Li}_2(1) - \ln x \ln(1-x) \tag{A.2}$$

$$\mathrm{Li}_2(x) + \mathrm{Li}_2(-x) = \frac{1}{2}\mathrm{Li}_2(x^2) \tag{A.3}$$

$$\mathrm{Li}_2(x) + \mathrm{Li}_2\left(\frac{-x}{1-x}\right) = -\frac{1}{2}\ln^2(1-x) \tag{A.4}$$

The golden mean (37) plays a special role in these identities. Setting $x = 1/\tau$ in the first two identities, we obtain

$$\frac{3}{2}\mathrm{Li}_2(1/\tau^2) - \mathrm{Li}_2(-1/\tau) = \mathrm{Li}_2 - \frac{1}{2}\ln\tau^2 \tag{A.5}$$

Setting $x = 1/\tau^2$ in the third identity, we obtain

$$\mathrm{Li}_2(1/\tau^2) - \mathrm{Li}_2(-1/\tau) = -\frac{1}{8}\ln^2\tau^2 \tag{A.6}$$

Combining Eqs. (A.5) and (A.6), we finally obtain

$$\mathrm{Li}_2(1/\tau^2) = \frac{2}{5}\mathrm{Li}_2(1) - \frac{1}{4}\ln^2\tau^2 \tag{A.7}$$

i.e., the dilogarithm can be expressed in terms of elementary functions at the special point $x = 1/\tau^2$. Eq. (40) then easily follows from (18) with M given by eq. (32).

Similarly, for the trilogarithm, the following identities hold:

$$\mathrm{Li}_3(x) + \mathrm{Li}_3(1-x) + \mathrm{Li}_3\left(\frac{-x}{1-x}\right) = \mathrm{Li}_3(1) + \mathrm{Li}_2(1)\ln(1-x) - \frac{1}{2}\ln x \ln^2(1-x) + \frac{1}{6}\ln^3(1-x) \tag{A.8}$$

$$\mathrm{Li}_3(x) + \mathrm{Li}_3(-x) = \frac{1}{4}\mathrm{Li}_3(x^2) \tag{A.9}$$

Again, the golden mean plays a special role. To see this, set $x = 1/\tau^2$ in the first identity,

$$\mathrm{Li}_3(1/\tau^2) + \mathrm{Li}_3(1/\tau) + \mathrm{Li}_3(-1/\tau) = \mathrm{Li}_3(1) + \frac{1}{2}\mathrm{Li}_2(1)\ln\tau^2 - \frac{5}{48}\ln^3\tau^2 \tag{A.10}$$

Using the second identity, we obtain

$$\mathrm{Li}_3(1/\tau^2) = \frac{4}{5}\mathrm{Li}_3(1) - \frac{2}{5}\mathrm{Li}_2(1)\ln\tau^2 - \frac{1}{12}\ln^3\tau^2 \tag{A.11}$$

which, in view of Eq. (A.7), can be written as

$$\mathrm{Li}_3(1/\tau^2) + \mathrm{Li}_2(1/\tau^2)\ln\tau^2 - \frac{1}{6}\ln^3\tau^2 = \frac{4}{5}\mathrm{Li}_3(1) \tag{A.12}$$

Eq. (39) follows from eq. (15) (with M given by eq. (32)) and eq. (A.12).

References

[1] M. R. Douglas, I. R. Klebanov, D. Kutasov, J. Maldacena, E. Martinec and N. Seiberg, "A new hat for the c = 1 matrix $O(N)$ Vector Model," arXiv:hep-th/0307195.

[2] T. Takayanagi and N. Toumbas, "A matrix $O(N)$ Vector Model dual of type 0B string theory in two dimensions," JHEP **0307**, 064 (2003) [arXiv:hep-th/0307083].

[3] P. H. Ginsparg and G. W. Moore, "Lectures on 2-D gravity and 2-D string theory," arXiv:hep-th/9304011.

[4] I. R. Klebanov, "String theory in two-dimensions," arXiv:hep-th/9108019.

[5] J. Polchinski, "What is string theory?," arXiv:hep-th/9411028.

[6] S. Alexandrov, "Matrix quantum mechanics and two-dimensional string theory in non-trivial backgrounds," arXiv:hep-th/0311273.

[7] Y. Nakayama, "Liouville field theory: A decade after the revolution," Int. J. Mod. Phys. A **19**, 2771 (2004) [arXiv:hep-th/0402009].

[8] E. J. Martinec, "The annular report on non-critical string theory," arXiv:hep-th/0305148.

[9] J. McGreevy and H. L. Verlinde, "Strings from tachyons: The c = 1 matrix reloaded," JHEP **0312**, 054 (2003) [arXiv:hep-th/0304224].

[10] V. A. Kazakov and A. A. Migdal, "Recent Progress In The Theory Of Noncritical Strings," Nucl. Phys. B **311**, 171 (1988).

[11] D. J. Gross and N. Miljkovic, "A Nonperturbative Solution Of D = 1 String Theory," Phys. Lett. B **238**, 217 (1990).

[12] D. J. Gross and I. R. Klebanov, "One-Dimensional String Theory On A Circle," Nucl. Phys. B **344**, 475 (1990).

[13] E. Brezin, V. A. Kazakov and A. B. Zamolodchikov, "Scaling Violation In A Field Theory Of Closed Strings In One Physical Nucl. Phys. B **338**, 673 (1990).

[14] P. H. Ginsparg and J. Zinn-Justin, "2-D Gravity + 1-D Matter," Phys. Lett. B **240**, 333 (1990).

[15] P. Hořava and C. A. Keeler, "Noncritical M-theory in 2+1 dimensions as a nonrelativistic Fermi liquid," arXiv:hep-th/0508024.

[16] G. Moore, "Double-scaled field theory at $c = 1$," Nucl. Phys. B **368**, 557 (1992).

[17] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, Academic Press, 1994.

[18] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, 2nd ed. Clarendon Press, Oxford, 1993.

[19] B. Rosenstein, B. Warr and S. H. Park, "Dynamical Symmetry Breaking In Four Fermi Interaction $O(N)$ Vector Models," Phys. Rept. **205**, 59 (1991).

[20] A. C. Petkou and M. B. Silva Neto, "On the free energy of three-dimensional CFTs and polylogarithms," Phys. Lett. B **456**, 147 (1999) [arXiv:hep-th/9812166].

[21] H. R. Christiansen, A. C. Petkou, M. B. Silva Neto and N. D. Vlachos, "On the thermodynamics of the 2+1 dimensional Gross-Neveu model with complex chemical potential," Phys. Rev. D **62** (2000) 025018 [arXiv:hep-th/9911177].

[22] S. Huang and B. Schreiber, "Statistical mechanics of relativistic anyon - like systems," Nucl. Phys. B **426** (1994) 644.

[23] W. Nahm, A. Recknagel and M. Terhoeven, "Dilogarithm identities in conformal field theory," Mod. Phys. Lett. A **8**, 1835 (1993) [arXiv:hep-th/9211034].

[24] L. Lewin, *Polylogarithms and Associated Functions*, North Holland, New York, 1981.